# Swaying Motion in Thermal Plume Above a Horizontal Line Heat Source

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**The swaying spectrum, the swaying frequency, and several characteristics in the swaying motion of a laminar thermal plume of air arising from a horizontal line heat source were experimentally investigated in an enclosure of 800 × 800 mm² square cross section and 1000 mm high. Temperature waves of swaying motion are apparently irregular. However, a certain kind of regularity exists in the probability density function and the autocorrelation coefficient of swaying temperature waves. Therefore, the spectrum analysis is carried out on the swaying temperature. As a result, the swaying spectrum has the slope values of −4.5 and −8.0 in the power spectrum density-frequency diagram. These slopes values −4.5 and −8.0 are the characterizing values of the laminar plume. Nondimensional formulas are obtained for swaying spectrum with the slope values of −4.5 and −8.0. These equations are useful for determination of the laminar region of the plume. The swaying frequency is proportional to ⅓ power of the heat rate from the line heat source. Nondimensional formulas of the swaying frequency are obtained with two kinds of representative lengths, *A* and *B*, as follows. $Ref_A = 0.333Ra_A^{1/3}$, $Ref_B = 0.219Ra_B^{1/3}$. Here, *Ref* and *Ra* are the frequency Reynolds and the Rayleigh numbers, respectively. By using these relations, it is possible to predict the swaying frequency of a plume in an enclosure with any dimensions.**

## Nomenclature

| | |
|---|---|
| $A$ | = distance from line heat source to ceiling, mm |
| $B$ | = distance from line heat source to side wall, mm |
| $d$ | = diameter of line heat source, mm |
| $f$ | = frequency, Hz |
| $Gr$ | = Grashof number, $L^3 g\beta Q/\lambda Pr \nu^2$ |
| $g$ | = gravitational acceleration, m/s² |
| $h(0)$ | = eigenvalue of nondimensional temperature |
| $I$ | = electric currents flowing in line heat source, $A$ |
| $i$ | = ordinal number of time series data |
| $L$ | = representative length, mm |
| $l$ | = length of line heat source, mm |
| $N$ | = sampling number |
| $P$ | = power spectrum density, $K^2 \cdot s$ |
| $Pr$ | = Prandtl number |
| $p$ | = probability density function, 1/K |
| $Q$ | = convective heat from line heat source, W/m |
| $Q_r$ | = radiative heat from line heat source, W/m |
| $Q_t$ | = generation of heat from line heat source, W/m |
| $R$ | = autocorrelation coefficient |
| $Ra$ | = Rayleigh number, $L^3 g\beta Q/\lambda\nu^2$ |
| $Ref$ | = frequency Reynolds number, $fL^2/\nu$ |
| $S$ | = power spectrum density normalized by temperature variance, s; $P/\sigma^2$ |
| $T$ | = time-averaged temperature, K; $(\Sigma t_i)/N$ |
| $T_w$ | = temperature of line heat source, K |
| $t$ | = instantaneous temperature, K; $T + t'$ |
| $t'$ | = variant temperature, K; $t - T$ |
| $V$ | = voltage difference between both ends of line heat source, V |
| $x$ | = vertical coordinate from line heat source, mm |
| $y$ | = horizontal coordinate perpendicular to line heat source, mm |
| $Z$ | = aspect ratio of enclosure, $A/B$ |
| $z$ | = axial coordinate of line heat source, mm |
| $\beta$ | = thermal coefficient of expansion, 1/K; $1/T_{\infty 2}$ |
| $\epsilon$ | = emissivity of nichrome wire, 0.7 |
| $\theta$ | = nondimensional temperature $(T - T_{\infty 2})/(T_{cl} - T_{\infty 2})$ |
| $\lambda$ | = thermal conductivity, W/m K |
| $\nu$ | = kinematic viscosity, m²/s |
| $\xi$ | = nondimensional distance of $y$ direction, $(y/x)Grx^{1/5}$ |
| $\sigma$ | = standard deviation of temperature, K; $[(1/N)\Sigma(t_i - T)^2]^{1/2}$ |
| $\sigma_s$ | = Stefan-Boltzman constant, W/m² K⁴ |
| $\tau$ | = time lag, s |

*Subscripts*

| | |
|---|---|
| $A$ | = for $L = A$ |
| $B$ | = for $L = B$ |
| $cl$ | = on midplane |
| $i$ | = for $i = i$ |
| $x$ | = for $L = x$ |

## Introduction

A THERMAL plume sways above a horizontal line heat source.[1-13] A history of investigations on the swaying motion of the plume above a line heat source is as follows.

In 1967, Forstrom and Sparrow[1] first discovered a swaying motion in the plume above a line heat source. In 1970, Schorr and Gebhart[2] confirmed the existence of swaying motion in a plume by flow visualization. In 1973, Fujii et al.[3] also observed the swaying motion of a plume by flow visualization. Later, investigations were carried out on plume swaying motions in the comparatively large cylinder by Akiyama[4] and in the comparatively small enclosure by Igarashi and Kada[5] and Igarashi.[6] Furthermore, the swaying frequency was obtained by Eichhorn and Vedhanayagam[7] in a water plume that interferes with the free surface between water and air. Urakawa et al.[8] showed that the plume in the spindle oil not only sways but also meanders to the axial direction of the heat source. The measurements by Yoshinobu et al.[9] indicated that the air plume also sways above a horizontal line heat source in the space enclosed with mesh walls. The transition from a laminar state to a turbulent one in plume has been investigated by Forstrom and Sparrow[1] and Bill and Gebhart.[10] Noto et al.[11] clarified the transition and the turbulent states in the plume. A swaying motion of the laminar and turbulent plume in the stably stratified fluid was experimentally investigated by Noto et al.[12,13]

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The swaying motion of a laminar plume in a comparatively large enclosure must be one of the most inherent and most fundamental motions of plume, because a laminar plume in a comparatively large enclosure is not influenced by the disturbances in the ambient fluid, walls of an enclosure, a free surface, and stratified temperature. Akiyama[4] treated the swaying motion of plumes inside a horizontal cylinder with comparatively large diameter. However, Akiyama could not obtain a characteristic of the swaying motion. Thus, characteristics and details of the swaying motion of plumes in a comparatively large enclosure have not been made clear.

Therefore, in this investigation, the swaying phenomenon in a laminar air plume above a horizontal line heat source inside a comparatively large enclosure was experimentally investigated. An aim of this investigation was to quantitatively clarify the swaying motion of a laminar air plume. Visual observations of swaying plumes were carried out. Time-dependent temperatures in the plume were measured and statistical analyses were done on time series data of plume temperatures. A swaying spectrum and a swaying frequency were obtained. Results obtained in this investigation are compared with previous experimental studies.

## Experimental Apparatus and Methods

The fluid used in this experiment is air at atmospheric pressure. An enclosure made of acrylic resin $800 \times 800$ mm$^2$ square in cross section and 1000 mm high was used. A horizontal line heat source for generating a plume was placed at a position 400 mm above the bottom surface. The line heat source was made of a nichrome wire, 0.435 mm in diameter and 550 mm long. The line heat source was electrically heated. A schematic view of the experimental apparatus and measurement system is shown in Fig. 1. Temperatures of the ceiling, the bottom surface, and the four side walls were isothermally controlled. The chromel-alumel thermocouples with $25.0 \sim 100.0\ \mu$ diam were employed for measurements of the transient plume temperature. The time constant of this thermocouple was larger than 25 Hz. This time constant is sensitive and reasonable for time-dependent measurement of temperature, spectrum analysis, and discussion of plume phenomenon, because the temperature frequency of the plume was smaller than 1.0 Hz. The output from the thermocouple reaches the pen recorder and the microcomputer. Statistical analyses were carried out by the microcomputer system. The sampling time and the sampling number for digitized temperature values were determined, as discussed in Ref. 14. The Nyquist frequency in the present sampling process is much larger than the plume frequency. The autocorrelation coefficient, the probability density function (PDF), and the power spectrum density (PSD) of swaying motion were obtained by the maximum entropy method (MEM).[15,16]

Convective heat transfer from the nichrome wire was obtained by the following equation:

$$Q = Q_t - Q_r \tag{1}$$

where

$$Q_t = IV/l, \qquad Q_r = \epsilon \sigma_S \pi d (T_w^4 - T_{\infty 1}^4) \tag{2}$$

Physical properties were determined under the following reference temperature:

$$T_r = T_{cl} - 0.38(T_{cl} - T_{\infty 2}) \tag{3}$$

where $T_{\infty 1}$ is the temperature at the wall of the laboratory room where the enclosure was placed. The radiant heat reaches the laboratory walls through the enclosure walls made of acrylic resin. $T_{\infty 2}$ is the temperature in the region of $y = 150 \sim 210$ mm in the enclosure, where the temperature is constant at the given height.

The plume phenomenon is more sensitive to temperature fluctuations outside the enclosure. Therefore, by observing the temperature of plume by the schlieren method, it was confirmed that temperature disturbances do not exist outside the enclosure. A reproducibility of outputs from the thermocouple was also confirmed. Measurements were carried out 1 h after the change of heat output from the nichrome wire. The experiment was carried out during the night while the atmospheric temperature did not change.

A flow visualization of the plume was carried out by applying a light sheet to the enclosure filled with smoke. A laminar plume transits to a turbulent one with an increase in the distance from the line heat source. Plume fluid circulates in the enclosure. Therefore, particles of smoke mix. As a result, a method filled with smoke in the enclosure was developed for the flow visualization of a plume. A streak line of a plume is observed as a black color on a photograph.

## Results and Discussion

### Visualization of Swaying Plume

First, we studied whether a plume meanders to the axial direction of the wire by flow visualizations. It was found that the plume does not meander to the $z$ direction in this experiment. Therefore, a thermofluid state of laminar plume is in a two-dimensional state. As a result, plume measurements were carried out on the plane of $z = 275$ mm. If the width of the

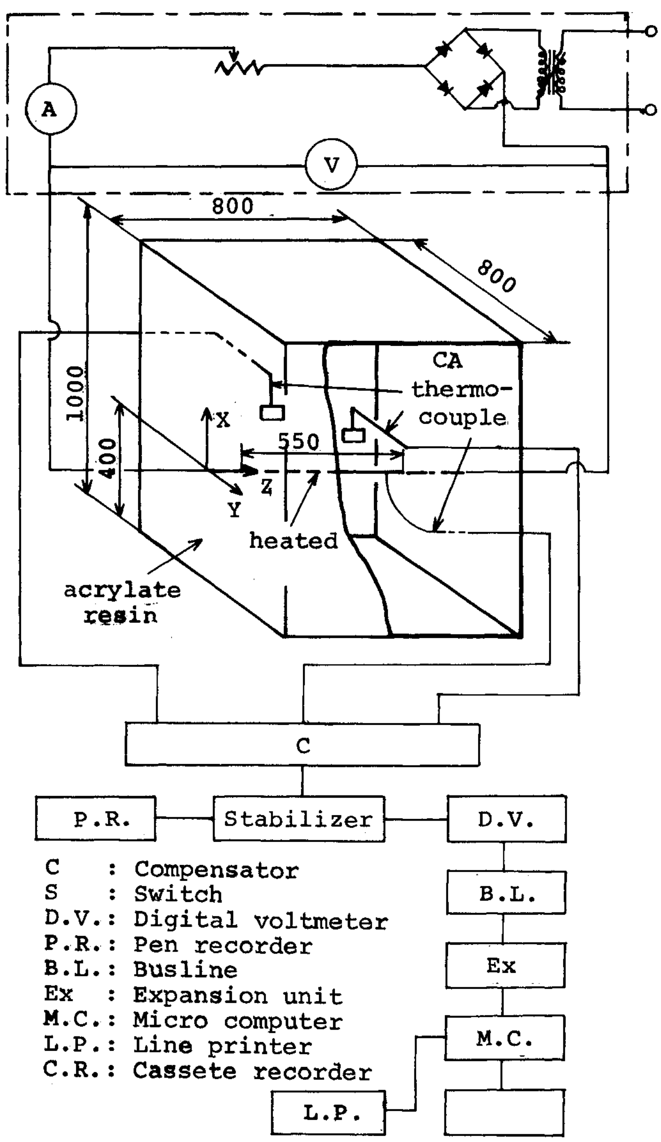



**Fig. 1 Experimental apparatus and measurement system.**

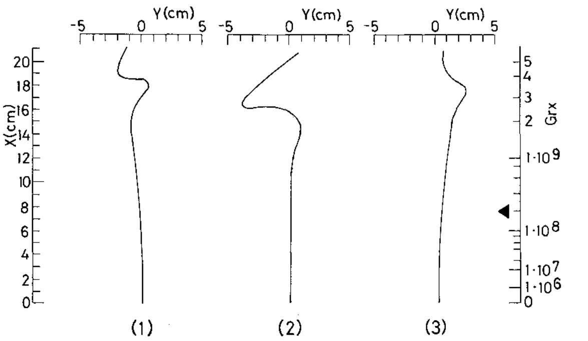
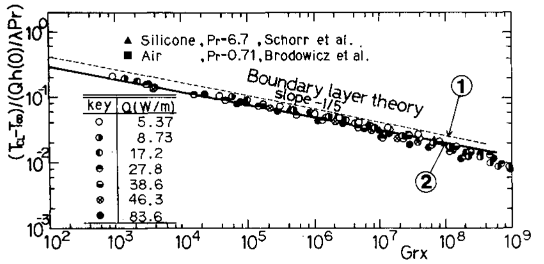
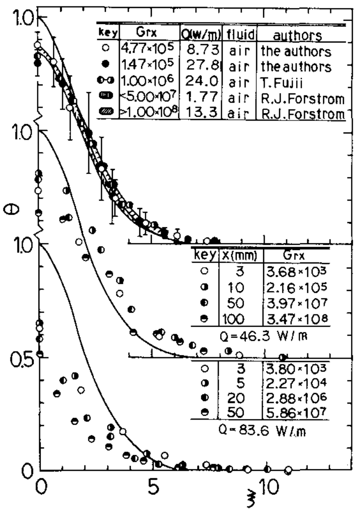



Fig. 2 Flow visualization of a swaying plume ($Q$ = 83.6 W/m).

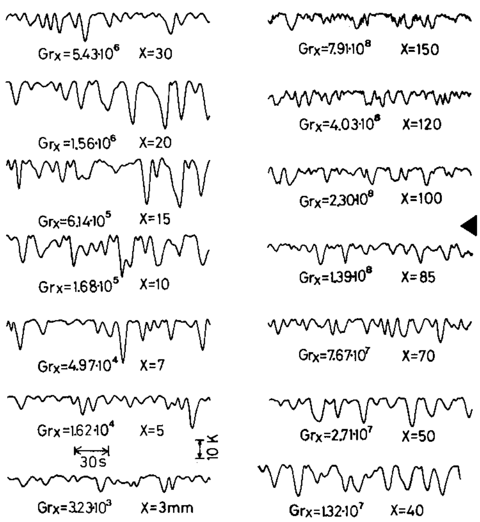



a) On midplane

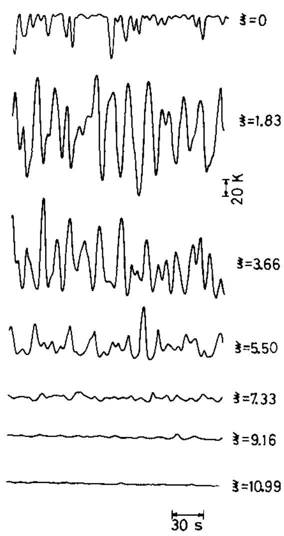



b) On section with given height

Fig. 3 Time-averaged temperature of a swaying plume.




a) On midplane ($Q$ = 33.7 W/m)




b) On section with given height ($Q$ = 83.6 W/m, $x$ = 3 mm, $Grx$ = 3.80 × 10$^3$)

Fig. 4 Temperature wave in a swaying plume.

enclosure becomes quite smaller than that of this experimental enclosure, a plume meanders to the $z$ direction.[17]

An example of flow visualization results of the plume is shown in Fig. 2. The line heat source is at $(x,y) = (0,0)$. The $x$ coordinate and the Grashof number are shown as an ordinate. The arrowhead indicates the location at the beginning of turbulent transition determined by the spectrum analysis, which is discussed later. The whole plume region sways from side to side. Only the time is different among Figs. 2a-2c. The plume in Fig. 2 is swaying with a period of about 15 s.

Plume temperature distributions were observed by the schlieren method. An instantaneous view of the plume swaying from side to side was observed. The swaying periods are about 25, 20, and 15 s for the heat outputs of 33.7, 46.3, and 86.3 W/m, respectively. Every plume bends at a certain point in the region $Grx > 1 \times 10^9$. With increasing the heat output from the line heat source, the plume bends at the position with a small value of $x$.

### Time-Averaged Temperature

A temperature distribution in the midplane ($y = 0$) is shown in Fig. 3a. The dotted line 1 indicates the result of the steady and laminar boundary-layer theory[18] without considering the fluid entrainment from $x < 0$ and the swaying motion. Experimental values can be approximated by the solid line 2. The slopes of lines 1 and 2 are both $-1/5$. Previous experimental results by Forstrom and Sparrow,[1] Schorr and Gebhart,[2] and Brodowicz and Kierkus[19] are in good agreement with the present results. The experimental distribution 2 is 15% lower than the theoretical distribution 1. This 15% difference is considered to be due to the fluid entrainment from $x < 0$.[20] The tendency for $Grx > (1 \sim 2) \times 10^8$ is different from that for $Grx < (1 \sim 2) \times 10^8$. This difference is caused by the transition from laminar to turbulent flow in the plume. The critical Grashof number at the beginning of the transition is discussed later. From the time-averaged temperature in the midplane, a laminar region exists for $Grx < (1 \sim 2) \times 10^8$.

The time-averaged temperature in the $y$ direction is shown in Fig. 3b. The value of $T_{cl}$ in the denominator of the ordinate $\theta$ is the theoretical value by the boundary-layer theory. The abscissa $\xi$ denotes the nondimensional similarity variable in the $y$ direction by the boundary-layer theory. The time-averaged temperature in the $y$ direction is symmetrical with respect to the plane of $y = 0$. Therefore, only the distribution for the region $\xi \geq 0$ is shown in Fig. 3b. The solid line indicates the theoretical value from the laminar boundary-layer theory. In the top figure, for the comparatively small heat output, the standard deviation is shown by a straight line. Previous experimental results by Forstrom and Sparrow[1] and Fujii et al.[3] are in good agreement with the present results. However, for a large heat output from the line heat source, the boundary-layer theory is not in agreement with the present results. This difference is discussed later.

**Temperature Wave of Swaying Plume**

Waves of temperature fluctuation at symmetrical points with respect to the midplane at the given height had a minus correlation. That is, an antisymmetry exists in temperature waves of a swaying plume. However, temperature waves are not apparently regular in all conditions of the present experiment. This is because of the following characteristics of the present experiment. The laminar plume transits to a turbulent one. The plume collides with the ceiling of the enclosure, then the plume fluid flows along the enclosure wall. In the meantime, the turbulence in the plume fluid diminishes due to the quite small velocity. As a result, the plume fluid circulates in the enclosure and is entrained into the plume.

An example of the temperature wave in the midplane is shown in Fig. 4a. The amplitude becomes larger with an increase of $x$. With further increase of $x$, the amplitude decreases. Every temperature wave is not apparently regular. Temperature waves for $Q = 5.37 \sim 83.6$ W/m were obtained. As a result, it can be concluded that the temperature fluctuation of a laminar plume increases and the amplitude of fluctuation becomes larger with an increase in heat output. For $Grx > (1 \sim 2) \times 10^8$, with an increasing Grashof number, fluctuations with a short period and high-frequency waves occur. With further increases of $x$, high-frequency waves become dominant.

An example of temperature waves for the different value of $y$ at a given height is shown in Fig. 4b. At $x = 3$ mm, the amplitude of the swaying wave becomes maximum at a position a little apart from the midplane. The amplitude of the temperature wave decreases in the $y$ direction from the point mentioned previously. Although not shown, at $x = 20$ mm, with an increase of $y$ the amplitude of the temperature wave is constant from the midpoint to a certain point and then decreases in the $y$ direction. At $x = 50$ mm, the amplitude of swaying wave decreases to the $y$ direction from the midplane. These characteristics are similar in every condition of the different outputs from that of Fig. 4b. As a result, it is found that the following three modes exist in the variation of the temperture wave amplitude.

1) Mode 1: The amplitude increases from the midplane and then decreases in the $y$ direction.

2) Mode 2: The amplitude is constant near the midplane and then decreases in the $y$ direction.

3) Mode 3: The amplitude decreases in the $y$ direction from the midplane.

With an increase of $x$, the amplitude mode changes from mode 1 to mode 3 through mode 2. With increasing heat output, the height (i.e., the downstream distance $x$) at which mode 2 or 3 occurs approaches the line heat source.

As shown in Figs. 4a and 4b, the waveform on the midplane is like a relaxation oscillation with a constant maximum temperature. On the other hand, the waveform has a constant minimum temperature in the region with a large value of $y$.

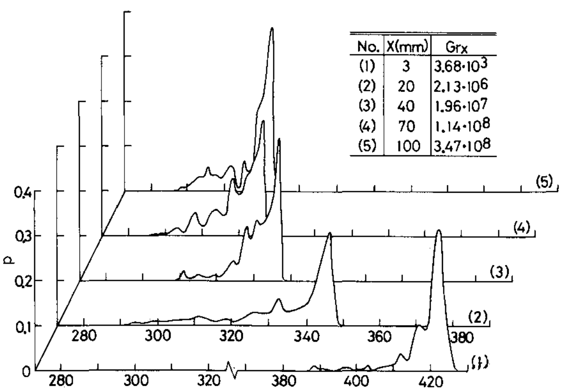



**Fig. 5 Probability density function ($Q = 46.3$ W/m).**

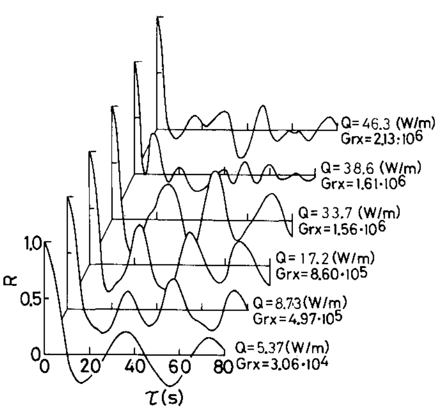



**Fig. 6 Autocorrelation coefficient ($x = 20$ mm).**

**Probability Density Function and Autocorrelation Coefficient**

A distribution of the PDF on the midplane is shown in Fig. 5. The abscissa is the instantaneous temperature and the ordinate the PDF. The PDF distribution is not symmetrical, because the waveform of the swaying plume is a relaxation oscillation. Therefore, the maximum value of the PDF exists at the higher temperature than at the mean temperature.

An example of the distribution of the autocorrelation coefficient on the midplane at a given height with different heat outputs is shown in Fig. 6. A strong correlation does not exist in every condition. However, the distribution of the autocorrelation coefficient does not converge to zero. That is, a weak correlation exists in every condition. Therefore, the fluctuation of the laminar swaying plume is not a random fluctuation but has a certain kind of regularity. The period of the autocorrelation coefficient decreases with an increase in the heat output. This fact corresponds to the decrease of the swaying peirod with an increase of heat output.

**Swaying Spectrum**

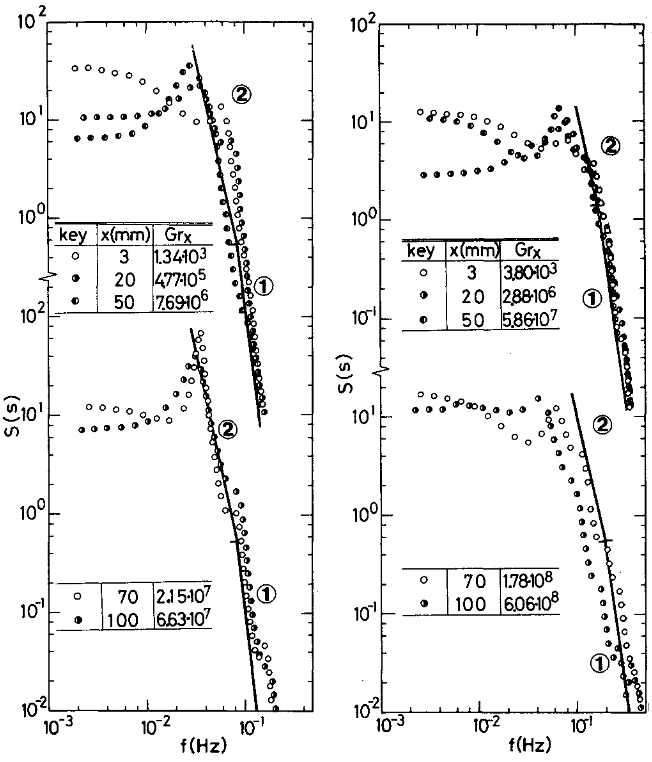
A spectrum analysis was carried out on swaying temperatures, because the swaying motion of the laminar plume has a certain kind of regularity.

The power spectrum densities on the midplane at the different heights are shown in Fig. 7. The ordinate is the power spectrum density normalized by the temperature variance. From Fig. 7, it is found that the swaying motion has several kinds of frequency. That is, the swaying spectrum is not a line spectrum but is a continuous spectrum. This fact corresponds to the result of temperature waveforms in the swaying motion. The slope value of $-4.5$ exists in every condition. In the higher frequency region, the slope value is equal to $-8.0$. When the heat output increases, the spectrum distribution shifts to the region with higher frequency. However, in this case, the slope values of $-4.5$ and $-8.0$ exist. That is, the spectrum slope values of $-4.5$ and $-8.0$ exist on the midplane.

The spectrum distributions for different values of $y$ are shown in Fig. 8. The slope values of $-4.5$ and $-8.0$ also exist.

Relations between $S$, $Q$, and $f$ are expressed for slope values of $-4.5$ and $-8.0$ by the following equations:

$$S = 6.47 \times 10^{-7}\, Q^{1.5} f^{-4.5}\ (1.51 \times 10^{-2} Q^{1/3} \le f \le 4.22 \times 10^{-2} Q^{1/3},\quad 5.37 \le Q \le 83.6) \tag{4}$$



a) $Q = 8.73$ W/m b) $Q = 83.6$ W/m

**Fig. 7 Power spectrum density of swaying motion (on midplane).**

$$S = 1.00 \times 10^{-11}\ Q^{8/3} f^{-8.0}\ (4.22 \times 10^{-2} Q^{1/3} \le f,$$
$$5.37 \le Q \le 83.6) \tag{5}$$

From Eqs. (4) and (5), the spectrum value increases in proportion to 1.5 and 8/3 power of the heat output. The solid lines in Figs. 7 and 8 represent distributions of Eqs. (4) and (5). In Fig. 7, Eqs. (4) and (5) are in good agreement with spectrum values for $Grx \le 1.78 \times 10^8$. For $Grx > 1.78 \times 10^8$, Eqs. (4) and (5) are not in agreement with spectrum values. This is due to the occurrence of turbulence in the plume. Therefore, the critical Grashof number is approximately equal to $Grx = 1.78 \times 10^8$. Equations (4) and (5) are also in good agreement with spectrum values in Fig. 8. All spectrum distributions for $Grx < 1.78 \times 10^8$ at different values of $y$ are in good agreement with Eqs. (4) and (5).

All of the 260 spectrum distributions in this experiment have slope values of $-4.5$ and $-8.0$ and are in good agreement with Eqs. (4) and (5).

Slope values of $-4.5$ and $-8.0$ also exist in the spectrum distribution of a laminar swaying plume in a thermally stratified fluid.[12,13] Therefore, it can be concluded that slope values of $-4.5$ and $-8.0$ are the characterizing values of the laminar swaying plume.

The value of $Grx = 1.78 \times 10^8$ is in good agreement with $Grx = (1 \sim 2) \times 10^8$ obtained from the midplane temperature in Fig. 3a. Therefore, the critical Grashof number at the beginning of transition is $Gr_{crit} = (1 \sim 2) \times 10^8$ in this experiment. The transition of a plume from laminar to turbulent flow is not drastic, because the plume transition is slow and natural. As a result, it becomes quite difficult to specify the transition of a plume as one point, as mentioned earlier. Forstrom and Sparrow[1] and Bill and Gebhart[10] obtained $Gr_{crit} = 5.8 \times 10^8$ and $1.12 \times 10^9$, respectively. The discrepancies in the critical Grashof number are due to the results of different methods used in its determination, i.e., the first appearance of turbulent bursts by Forstrom and Sparrow, the visual observation by Bill and Gebhart, and the PSD slope criterion by this investigation.

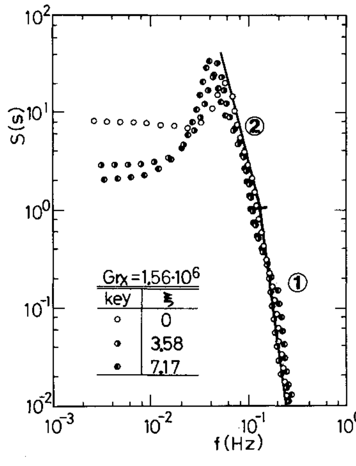
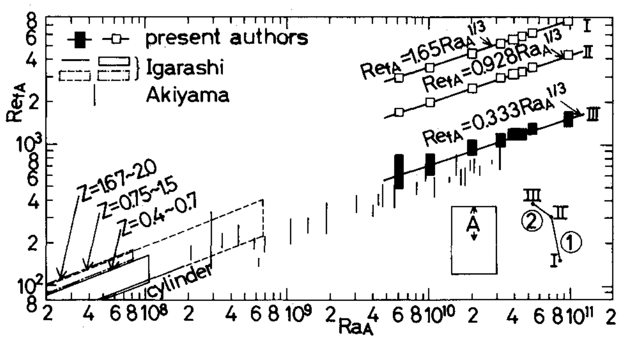
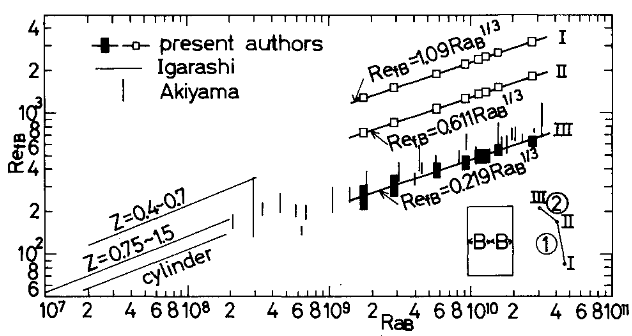


**Fig. 8 Power spectrum density of swaying motion (on section with given height, $Q = 33.7$ W/m, $x = 20$ mm).**



a) For representative length $A$



b) For representative length $B$

**Fig. 9 Swaying frequency of plume.**

The thermofluid state for $Grx < Gr_{crit}$ is in the laminar state. Therefore, the discrepancy in Fig. 3b between experimental results and the boundary-layer theory is not due to turbulence but to laminar phenomenon disregarded in the laminar boundary-layer theory.

### Swaying Frequency

Relations between the swaying frequency $f$ and the convective heat transfer $Q$ from the line heat source are shown in Fig. 9. The swaying plume reaches the ceiling of the enclosure in a laminar or a turbulent state. On the other hand, the onset of the meandering phenomenon in the plume is determined by the width of the enclosure. Therefore, it is necessary to choose the distances $A$ from the wire to the ceiling and $B$ from the wire to the side wall as a representative length.

Point I in Fig. 9 denotes the location where the value of the spectrum $S$ is equal to $1 \times 10^{-2}$. Point II denotes the location where the $S$ value obtained by Eq. (4) is equal to that obtained by Eq. (5). Frequency distributions at points I and II are expressed by the following relations:

Point I:

$$Ref_A = 1.65 Ra_A^{1/3}, \qquad Ref_B = 1.09 Ra_B^{1/3} \tag{6}$$

Point II:

$$Ref_A = 0.928 Ra_B^{1/3}, \qquad Ref_B = 0.611 Ra_B^{1/3} \tag{7}$$

Distribution of the most dominant frequency is shown by the filled rectangles in Fig. 9. Distributions of the most dominant frequency can be approximated by the following expressions. The frequency Reynolds number is proportional to one-third power of the Rayleigh number:

$$Ref_A = 0.333 Ra_A^{1/3} \quad (6 \times 10^9 \le Ra_A \le 1 \times 10^{11})$$

$$Ref_B = 0.219 Ra_B^{1/3} \quad (2 \times 10^9 \le Ra_B \le 3 \times 10^{10}) \tag{8}$$

Distributions determined by Eqs. (6-8) were plotted in Fig. 9. The distribution of point I is considered a standard of frequency range of the swaying motion.

The swaying frequency determined by flow visualization was in good agreement with the most dominant frequency determined by Eq. (8). Therefore, it is reasonable to identify the most dominant frequency as the swaying frequency of the plume.

The nondimensional expressions of the PSD with the slope values of $-4.5$ and $-8.0$ are as follows:

$$P\nu/(\sigma A)^2 = \left\{ \begin{array}{l} 3.08 \times 10^{-5} Ra_A^{1.5} Ref_A^{-4.5} \\ \quad (0.333 Ra_A^{1/3} \le Ref_A \le 0.928 Re_A^{1/3}) \\ 2.37 \times 10^{-5} Ra_A^{8/3} Ref_A^{-8.0} \\ \quad (0.928 Ra_A^{1/3} \le Ref_A) \\ (6 \times 10^9 \le Ra_A \le 1 \times 10^{11}) \end{array} \right\} \tag{9}$$

$$P\nu/(\sigma B)^2 = \left\{ \begin{array}{l} 1.08 \times 10^{-5} Ra_B^{1.5} Ref_B^{-4.5} \\ \quad (0.219 Ra_B^{1/3} \le Ref_B \le 0.611 Ra_B^{1/3}) \\ 1.94 \times 10^{-6} Ra_B^{8/3} Ref_B^{-8.0} \\ \quad (0.611 Ra_B^{1/3} \le Ref_B) \\ (2 \times 10^9 \le Ra_B \le 3 \times 10^{10}) \end{array} \right\} \tag{10}$$

Previous experimental results[4-6] are also shown in Fig. 9. Akiyama did not carry out a spectrum analysis on the swaying temperature. Distributions were calculated from Fig. 9 of Ref. 4 and are plotted in Fig. 9. The aspect ratio of the enclosure in the present experiment is equal to 1.5. The meandering phenomenon was found to occur in the experiments of Refs. 5 and 6, because the width of the enclosure used was small.

The swaying frequency is proportional to one-third power of the Rayleigh number within the present experimental range. For small Rayleigh numbers ($Ra_A < 1 \times 10^8$), the swaying frequency is proportional to 0.4 power of the Rayleigh number. This difference in exponential values is because a plume reaches the ceiling in a laminar or turbulent state. A laminar plume transits to a turbulent one at $Grx = (1 \sim 2) \times 10^8$, and this Grashof number is equivalent to $Ra_A = 7 \times 10^7 \sim 1.4 \times 10^8$. The shift of the slope value of 0.4 to ⅓ in Fig. 9a occurs in the neighborhood of the previously mentioned Rayleigh number.

## Conclusions

The spectrum and the frequency of swaying motion in the thermal air laminar plume arising from a horizontal line heat source were experimentally investigated in an enclosure of $800 \times 800$ mm$^2$ square cross section and 1000 mm in height. This plume was in a two-dimensional state. The following conclusions were obtained:

1) Critical Grashof number for the beginning of transition: The $Gr_{\text{crit}}$ for the beginning of the transition is equal to $(1 \sim 2) \times 10^8$. This value was determined based on the slope value of the power spectrum density and the midplane temperature distribution.

2) Swaying temperature: The temperature wave of the swaying plume varies with time and is apparently irregular. However, the temperature wave in the midplane has a maximum constant value. In the circumference of the plume, a minimum constant value occurs in the temperature wave.

3) Swaying spectrum: At any location in the laminar region, frequency bands exist where the power spectrum density is proportional to $-4.5$ and $-8.0$ power of the frequency. These slope values of $-4.5$ and $-8.0$ are the characterizing values of laminar plume.

4) Distribution of swaying spectrum: Nondimensional relations of swaying spectrum are expressed in Eqs. (9) and (10) in regions with the slope values of $-4.5$ and $-8.0$, respectively. These equations are useful for determining the laminar region of the plume.

5) Swaying frequency: The swaying frequency of the thermal air plume in a comparatively large enclosure is proportional to ⅓ power of the heat rate of the line heat source.

In the present paper, the swaying motion of the laminar thermal plume was clarified. By using relations obtained in this investigation, one can designate the laminar region and predict the swaying spectrum and frequency of a plane plume in an enclosure with any dimensions. The results obtained in this investigation are extremely useful for interpreting swaying motions of plumes in a turbulent state and in a temperature stratified fluid.

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